14.11. Model: The air-track glider is in simple harmonic motion.

Solve: **(a)** We can find the phase constant from the initial conditions for position and velocity:

$$x_0 = A\cos\phi_0 \qquad v_{0x} = -\omega A\sin\phi_0$$

Dividing the second by the first, we see that

$$\frac{\sin\phi_0}{\cos\phi_0} = \tan\phi_0 = -\frac{v_{0x}}{\omega x_0}$$

The glider starts to the left ($x_0 = -5.0$ cm) and is moving to the right ($v_{0x} = +36.3$ cm/s). With a period of 1.5 s = $\frac{3}{2}$ s, the angular frequency is $\omega = 2\pi/T = \frac{4}{3}\pi$ rad/s. Thus

$$\phi_0 = \tan^{-1}\left(-\frac{36.3\ \text{cm/s}}{(4\pi/3\ \text{rad/s})(-5.0\ \text{cm})}\right) = \tfrac{1}{3}\pi\ \text{rad}\ (60°)\ \text{or} -\tfrac{2}{3}\pi\ \text{rad}\ (-120°)$$

The tangent function repeats every 180°, so there are always two possible values when evaluating the arctan function. We can distinguish between them because an object with a negative position but moving to the right is in the third quadrant of the corresponding circular motion. Thus $\phi_0 = -\frac{2}{3}\pi$ rad or –120°.

(b) At time t, the phase is $\phi = \omega t + \phi_0 = (\frac{4}{3}\pi\ \text{rad/s})t - \frac{2}{3}\pi\ \text{rad}$. This gives $\phi = -\frac{2}{3}\pi$ rad, 0 rad, $\frac{2}{3}\pi$ rad, and $\frac{4}{3}\pi$ rad at, respectively, $t = 0$ s, 0.5 s, 1.0 s, and 1.5 s. This is one period of the motion.